Convergence Tests for Series

Test for Divergence $\sum_{n=1}^{\infty} a_n$	▪ If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges ▪ If $\lim_{n\to\infty} a_n = 0$, then inconclusive
Geometric Series $\sum_{n=0}^{\infty} ar^{n-1}$	▪ If $\|r\| < 1$, the series converges to $\frac{a}{1-r}$ ▪ If $\|r\| \geq 1$, then the series diverges
Integral Test $\sum_{n=c}^{\infty} a_n$ where $c \geq 0$ and $a_n = f(n)$ for all n	▪ $f(n)$ must be continuous, positive, and decreasing ▪ If $\int_c^{\infty} f(x)dx$ converges, then the series converges ▪ If $\int_c^{\infty} f(x)dx$ diverges, then the series diverges
p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$	▪ If $p > 1$, then the series converges ▪ If $p \leq 1$, then the series diverges
Comparison Test $\sum a_n$ and $\sum b_n$ where $0 \leq a_n \leq b_n$ for all n	▪ If $\sum b_n$ converges, then $\sum a_n$ converges ▪ If $\sum a_n$ diverges, then $\sum b_n$ diverges
Limit Comparison Test $\sum a_n$ and $\sum b_n$ where $a_n, b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$	▪ If $\sum b_n$ converges, then $\sum a_n$ converges ▪ If $\sum a_n$ diverges, then $\sum b_n$ diverges ▪ To find b_n consider only the terms of a_n that have the greatest effect on the magnitude
Alternating Series Test $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ where $b_n > 0$	▪ Converges if $0 < b_{n+1} < b_n$ for all n and $\lim_{n\to\infty} b_n = 0$
Absolute Value Test $\sum a_n$	▪ If $\sum\|a_n\|$ converges, then $\sum a_n$ converges ▪ If the series of absolute values $\sum\|a_n\|$ is convergent, then the series is ***absolutely convergent*** ▪ If the series is convergent but not absolutely convergent, then the series is ***conditionally convergent***
Ratio Test $\sum a_n$ with $\lim_{n\to\infty} \frac{\|a_{n+1}\|}{\|a_n\|} = L$	▪ If $L < 1$, then the series converges absolutely ▪ If $L > 1$ or L is infinite, then the series diverges ▪ If $L = 1$, then the test is inconclusive
Root Test $\sum a_n$ with $\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = L$	▪ If $L < 1$, then the series converges absolutely ▪ If $L > 1$ or L is infinite, then the series diverges ▪ If $L = 1$, then the test is inconclusive

Flowchart for Convergence Tests for Series